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FINE SYNCHRONIZATION OF COHERENT FREQUENCY HOPPING SIGNALS

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Special Communications Branch
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A method for precise synchronization of receiver timing with a phase coherent frequency hopping signal has been developed. It is assumed that initial synchronization by other techniques has aligned the receiver frequency hopping sequence with the signal well enough so that the receiver operates on the same frequency as the signal for most of each hop period. Phase comparisons between the signal and a locally generated replica then can be made and summed over a large number of frequency hops to provide a sensitive measure of the remaining synchronization error.

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20. Abstract (Continued)

Frequency dependent errors in the phasing of quadrature channels in the experimental synchronizer produced ripple in the detector output. Additional ripple was produced by circuits used for combining the quadrature channel outputs. The observed ripple produced reversals in the slope of the error signal, and, in a complete system, would have led to limited instability of synchronization. Criteria have been developed for determining circuit characteristics necessary to reduce the ripple to an acceptable value.

The effects on the synchronization method of frequency offsets and of several modulation types have been examined, and techniques have been proposed for maintaining close synchronization in the presence of these effects.

CONTENTS

FINE SYNCHRONIZATION OF COHERENT FREQUENCY HOPPING SIGNALS

INTRODUCTION

A frequency hop communication system is one in which the signal carrier frequency is switched more or less rapidly among a number of assigned channels. The frequency switching rate and the order in which the frequencies are chosen are system parameters known to the system users, as is the nominal time of the start of the switching sequence. Clock drift and uncertainty as to the distance from the transmitter to the receiver introduce a time uncertainty that makes a synchronization process necessary to align the receiver frequency selection sequencer and other timing circuits with the received signal pattern. The accuracy of synchronization required, or possible, is dependent on system and signal characteristics. The synchronization process considered in this report is possible only when the r.f. phase of the signal is predictable for each signal frequency, and depends on matching a replica of the transmitted signal, generated in the receiver, to the received signal. This technique can permit synchronization accuracy proportional to the full system bandwidth for use when very accurate distance measurement or time transfer is necessary, and allows coherent demodulation of the signal. However, the practical difficulties of generating the coherent frequency-hopping signals and of synchronizing the receivers have so far prevented operational use of a coherent frequency hopping system. The technique has been demonstrated at NRL with coherent frequency synthesizers developed under contract N00012-69-C-0117. NRL work on the initial stages of synchronization (applicable to both coherent and non-coherent signals) has been reported in references 1 and 2.

SYNCHRONIZATION

The synchronization process can be divided into three stages, each more precise than the one preceding. The first two stages have been called acquisition and coarse synchronization; they have been discussed in detail in references 1 and 2, so only a brief description will be given here. Acquisition is a search for a recognizable signal pattern in a time frame determined by the estimated time error and propagation delay at the receiver. This search is initiated by advancing the local sequencer in time an amount slightly greater than the expected time uncertainty and then stopping it until a signal is received on the frequency the sequencer has stopped on. The sequencer is then started and the subsequent frequency selections are examined for the presence of signal. If signals are detected in a substantial number of the frequency channels examined, it is assumed that the sequence timing is approximately correct and that acquisition is

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complete. If signals are not observed regularly in the hops following the first detection, the sequencer is again stopped to await another signal. The process repeats until acquisition is achieved or it is necessary to reset the sequencer further ahead in time. The acquisition process can be expected to reduce the synchronization error to less than one half of a hop period. Following acquisition, a coarse synchronization process is started. This compares the amounts of signal present in two overlapping time periods to generate timing correction signals that reduce the error to a small fraction of a hop period. The acquisition and coarse synchronization stages of synchronization are limited in performance by their inability to coherently integrate the received signal for longer than one frequency hop period. This difficulty is overcome in the next stage of synchronization.

FINE SYNCHRONIZATION

Fine synchronization operates, after acquisition and coarse synchronization are complete, by examining the phase differences for each hop between the signal and a locally generated signal replica and adjusting the replica timing so that the phase difference becomes constant for all hops. An analog method of processing the phase difference measurements will be analysed, but several digital techniques are possible and would probably be investigated if development continued.

The condition at the start of fine synchronization is that the signal replica in the receiver has been brought very close to synchronization with the signal. If the replica and the signal are connected to a mixer (multiplier), followed by a low pass filter to remove the double frequency component, a series of d.c. levels will be obtained representing the phase differences between the signal and the replica at each frequency. The phase difference can be written:

$$\theta_i = t_e f_i, \tag{1}$$

where t_e is the synchronization error in seconds, f_i is the frequency in hertz of hop i i is an integer from 1 to n and θ_i is the phase difference in cycles between the signal and the replica. To simplify further analyses, the variation of phase difference from hop to hop will be used as the working variable. With θ_1 as the reference:

$$\phi_i = \theta_i - \theta_1 = t_e (f_i - f_1). \tag{2}$$

The interval between adjacent frequencies will ordinarily be equal to the system bandwidth, w, divided by the number of frequencies, n, so:

$$f_i - f_1 = (w/n)\ (i - 1) \text{ and} \tag{3}$$

$$\phi_i = t_e (w/n)(i - 1) \text{ cycles} \tag{4}$$

For a given synchronization error the output, ϕ_i, of the multiplier/low-pass-filter combination will consist of a series of

voltage levels with a pattern dependent on the frequency selection sequence. To obtain the full advantages of coherent operation, it is necessary to integrate levels representing phase differences at all n frequencies in developing the synchronizing signal. The result of the integration can be arrived at by considering each of the phase measurements, ϕ_i, as the direction of a vector of unit magnitude representing the relative phase difference at the i-th frequency. If the synchronization is perfect, $t_e = 0$ and all the vectors add in line to produce a resultant of magnitude n (if the number of vectors added is n). For $t_e \neq 0$, the vectors are ordinarily not in line and a smaller resultant is produced. The normalized magnitude of the sum of n unit magnitude vectors representing the phase differences ϕ_i is:

$$A = (1/n)\left[\left(\sum_{i=1}^{n} \cos \phi_i\right)^2 + \left(\sum_{i=1}^{n} \sin \phi_i\right)^2\right]^{\frac{1}{2}} \qquad (5)$$

$\phi_1 = 0$ by definition, $\phi_2 = t_e(w/n)$ and $\phi_i = (i - 1)\phi_2$ from eq (4), so:

$$A = (1/n)\left[\left(1 + \sum_{i=1}^{n-1} \cos i\phi_2\right)^2 + \left(\sum_{i=1}^{n-1} \sin i\phi_2\right)^2\right]^{\frac{1}{2}} \qquad (6)$$

From reference 5:

$$A = (1/n)\left[\left(1 + \frac{\sin \frac{1}{2}(n-1)\phi_2 \cos \frac{1}{2}n\phi_2}{\sin \frac{1}{2}\phi_2}\right)^2 + \left(\frac{\sin\frac{1}{2}(n-1)\phi_2 \sin \frac{1}{2}n\phi}{\sin \frac{1}{2}\phi_2}\right)^2\right]^{\frac{1}{2}} \qquad (7)$$

In equations (5) through (7) it is assumed that the n frequencies are evenly spaced in the band w and that all n vectors are added to obtain the sum A.

Figure 1 is a plot of A vs. t_e for a system with w = 12.8 MHz, hopping rates equal to w/n and n = 4, 32 and 256. The corresponding values of ϕ_2 are $3.2t_e \times 10^6$, $4t_e \times 10^5$ and $5t_e \times 10^4$ cycles. Curves for n greater than 256 are nearly indistinguishable from the n = 256 curve at the scale used for the figure. If the time scale were extended, the curves would eventually repeat, but the required extension is beyond the range of applicability for reasonably large n. Figure 2 shows the difference curves obtained by subtracting the outputs of two vector adders, one of which has had its input delayed d nanoseconds. The detector response for n = 256 was used in calculating these curves. The amplitudes of the curves increase with d until d = 1/w (the first null in Figure 1, curve 1); beyond that the peak amplitude decreases (see the d = 100 ns curve) and further increases in d would place the difference curve zero crossing too far down on the detector response curve to be acceptable. Values of d in the range from 60 ns to 100 ns appear to provide the best difference curves for a system with the specified 12.8 MHz bandwidth. If the useful range of response is taken to be between the half amplitude points (on the outsides of the peaks), then the values are: ±67 ns for d = 60 ns, ±74 ns for d = 80 ns and ±90 ns for d = 100 ns. If a

larger pull-in range is needed, use of less than the full system bandwidth will broaden the curves, or an additional delayed channel could be used.

The accuracy of synchronization will depend on the signal to noise ratio (S/N) and the integration time. The S/N power ratio can be expressed as $A^2/2\sigma^2$, where A is the peak signal voltage and σ is the rms noise voltage. Substraction of two detector outputs to obtain a difference characteristic such as those of Fiqure 2 does not increase the effective signal output, because of the offset of the two detector characteristics, but does allow the noise powers to add. The result is 3 dB reduction in S/N at the output of the difference circuit.

The synchronization error (1σ) corresponding to a given S/N can be determined by finding the relative noise voltage for the S/N, multiplying by 1.414 to account for the noise addition in the difference circuit, and scaling the synchronization error from Figure 2. The table lists values of synchronization error for several S/N; the d = 80ns curve in Figure 2 was used.

S/N (dB)	Effective Noise Voltage ($\sqrt{2}\,\sigma/A$)	Synchronization Error (1σ) (ns)
3	0.708	23.5
6	0.501	16.0
12	0.251	7.8
18	0.126	3.9

The detection curves used to obtain the table assumed integration of phase measurements at 256 equally spaced frequencies. Ideally, the input S/N to the integrator should be 10 log 256 (24dB) less than the S/N shown in the left hand column of the table. Probably at least a 6 dB post detection S/N would be desired for reliable synchronization, so an input S/N of a least -18 dB would be required.

Figure 3A is a block diagram of a synchronization detector using the technique just described. The signal is multiplied by four phases of the local reference and low-pass filtered (integrated) to produce in-phase (I) and quadrature (Q) components of the phase differences between the signal and the direct and delayed references. The I and Q components are represented by the cosine and sine summations, respectively, of equation (5). The integration time should be sufficient to include frequency hops occupying the entire band. Following the integrators are combiners to develop the magnitudes of the vector sums from the I and Q components in the direct and delayed channels. The output of the direct channel is as shown in Figure 1, and the subtraction of the output of the delayed channel from the direct channel output results in a difference curve such as one of those shown in Figure 2.

Figures 3B and 3C indicate two methods of combing the I and Q

channels. Figure 3C is the classical technique of squaring the components, adding the squares and taking the square root of the sum. This produces an accurate representation of the magnitude of the vector sum of I and Q, but requires, for this application, accurate analog squaring and square-rooting of d.c. and low frequency a.c. signals; alternatively, the I and Q values could be converted to digital form and processed digitally. Figure 3B is an approximation technique requiring only linear elements and switches to approximate the true magnitude within 8%.

EXPERIMENTAL RESULTS

An experimental synchronization detector was built according to the diagrams of Figures 3A and 3B, with the substitution of delay lines for the quadrature hybrids shown in Figure 3A. Operation was tested with the aid of two coherent frequency synthesizers operating with 12.8 MHz bandwidth centered at 70 MHz. The number of frequencies could be varied from 32 to 256, with corresponding hopping rates of 400 kH/s to 50 kH/s, respectively; most of the experimentation was done with the 50 kilohop per second rate and 256 frequencies. Results generally followed the expected form, and observation of the phase difference patterns ahead of the integrators clearly showed the phase differences approaching a common value as synchronism was approached. Figure 4 is a photograph of the difference curve produced by the experimental circuit. The general shape of the curve is correct, but there is a substantial ripple component that would interfere with synchronization in a working system. This ripple was determined to be a characteristic of the circuit design and not a result of the measurement technique or test equipment.

ERROR ANALYSIS

It appeared probable that the ripple was caused by departure from 90° phase separation of the quadrature channel inputs and/or by the approximations in the I and O combiners (Figure 3B). The delay lines used to separate the quadrature channels in phase were of length equivalent to 90° at the center frequency of the band, 70 MHz, so phase errors 6.4 MHz away at either edge of the band were $\pm 8.23^\circ$. The combiner circuits, as noted above, introduced amplitude variations of 8% peak-to-peak.

Analysis proceeded on the basis that, in the region near the center of the difference curve, the slope of the ripple component must be less than the slope of the desired detector output. The general expression for the output of a quadrature combining circuit is:

$$R = (I^2 + Q^2)^{\frac{1}{2}} \tag{8}$$

where I = cos a, Q = sin a and a is the phase angle between the signal and the local reference. Substituting and introducing an error angle,

b, divided equally between the I and Q channels yields:

$$R = [\sin^2(a-\tfrac{1}{2}b) + \cos^2(a+\tfrac{1}{2}b)]^{\frac{1}{2}} \tag{9}$$

This can be written more informatively as:

$$R = (1 - \sin 2a \sin b)^{\frac{1}{2}}, \tag{10}$$

from which it is evident that the maxima and minima of R occur when $\sin 2a = \pm 1$, i.e. when a is 45°, 135°, 225° and 315°. When $\sin 2a = \pm 1$,

$$R = (1 \mp \sin b)^{\frac{1}{2}} = \cos \tfrac{1}{2}b \mp \sin \tfrac{1}{2}b, \tag{11}$$

so R has a peak-to-peak amplitude variation of $2\sin \tfrac{1}{2}b$. The maximum slope of the ripple is $-2\sin \tfrac{1}{2}b$, and is derived in the appendix. It has a value, in the experimental system, of:

$$(dR/dt_e)_{max} = 0.8796 \sin \tfrac{1}{2}b \text{ units/nanosecond} \tag{12}$$

This slope must be kept smaller than the slope of the synchronization detector response in the region where the direct and delayed responses are equal. Table 1 lists the magnitudes and slopes of the detector responses for a number of synchronization errors (see also curve 1, Figure 1).

Table 1

Sync. Error t_e(ns)	Response Magnitude A	Response Slope dA/dt_e (1/ns)
20	0.896	0.0101
30	0.774	0.0140
40	0.621	0.0165
50	0.450	0.0175
60	0.276	0.0170

The difference curve is formed by subtracting the detector response to a delayed input from the detector response to an undelayed input. The width of the response curve, from peak to peak, is only slightly affected by the delay time, d, in the region of interest. The slope between the peaks is strongly affected by the delay, as is evident in Figure 2. The maximum amplitude of the difference response occurs when $d = 1/w$, which places the peak of one response over the null of the other; the $d = 30$ ns curve of Figure 2 approximates this. The center of the difference curve, the zero crossing, is labelled zero time in Figures 2 and 6; it occurs when the two detector outputs have equal amplitudes, which happens when the synchronization error is d/2 relative to both direct and delayed response peaks. Since the

interesting values of d are from 60 ns to 100 ns, the corresponding values of t_e are from 30 ns to 50 ns. The effective ripple amplitude and slope are products of the ripple parameters and the magnitude of detector response at the point under consideration, so the limits on the permissible quadrature error can be found from:

$$A(dR/dt_e)_{max} \leq dA/dt_e \tag{13}$$

For the experimental system:

$$0.8796\ A \sin \tfrac{1}{2}b \leq dA/dt_e \tag{14}$$

Table 2 lists limiting values of b for some of the synchronization errors of interest (values of A and dA/dt_e from Table 1):

Table 2

Sycn Error t_e (ns)	$A(dR/dt_e)_{max} \leq dA/dt_e$	b_{max}
30	$0.6813 \sin \frac{1}{2}b \leq 0.0140$	2.36°
40	$0.5465 \sin \frac{1}{2}b \leq 0.0165$	3.46°
50	$0.3959 \sin \frac{1}{2}b \leq 0.0175$	5.07°

Figure 5 is the calculated detector response vs. synchronization error for quadrature error, b, equal to 5°. From Table 2, this error should produce a ripple slope equal to the detector slope for t_e near 50 ns. Examination of the curve shows diminishing ripple amplitude as t_e increases, and from approximately 50 ns to the end of the plot there are no slope reversals. Figure 6 shows one half of the difference curves obtained for the 5° quadrature error and for various peak separations. In Figure 6, $t_e = 0$ corresponds to the center of the difference curve (one half of the peak separation from the zero time of Figure 5). A peak separation of 100 ns would put the center of the difference curve 50 ns from either peak, so the resulting response should be monotonic near the center. The figure confirms this. The curve for 50 ns peak separation should, and does, have excessive ripple - the zero error point has reverse slope and would be unstable, so the system would tend to synchronize with a $\pm$ 5 ns error. The other curves of Figure 6 show the effects of a range of peak separations (from 80 ns to 98 ns) on ripple content. The curve for peak separation of 96 ns shows nearly complete cancellation of the ripple near the center of the curve. This cancellation makes the curve nearly straight near the center, which would probably improve the stability of the synchronization process. It appears that the cancellation could be used to remove fairly large ripple components, provided that they were caused by stable quadrature, or other, errors.

The experimental synchronization detector had a difference curve peak separation of approximately 75 ns, so the maximum allowable ripple slope would be approximately 0.0159/ns. The corresponding peak-to-peak amplitude ripple is 5.5% and the quadrature error sufficient to produce the limiting slope is 3.14°. The 8% peak-to-peak amplitude variation of the approximate combiner and the $\pm 8.23^{\circ}$ quadrature errors of the Q channel reference delay lines are both substantially greater than the maximum allowed for monotonic difference response and, with some allowance for ripple cancellation effects, are sufficient to account for ripple observed in Figure 4.

Use of broadband quadrature hybrids instead of delay lines can reduce the quadrature error to approximately $\pm 2^{\circ}$ across the band. Precision analog or digital circuits can be used for combination of the I and Q channels as indicated in Figure 3C. The combined effect of these improvements and, possibly, increasing the difference curve peak separation to 90 to 100 ns should enable monotonic detection of synchronization error in future implementations of the synchronization detector.

MODULATION EFFECTS

In the synchronization process considered above, reception of an unmodulated signal has been assumed. It will be necessary in practice to maintain fine synchronization with modulated signals, so means must be found to accept various forms of modulation.

Time-gating or on-off keying does not interfere with the operation of the synchronizer of Figure 3 except for the loss in signal-to-noise ratio caused by unpredictable off time of the signal. The result will be less accurate synchronization unless longer integration times or more signal power are used.

Phase shift keying (PSK) is a more difficult problem. Since the synchronization process depends on addition of vectors representing phase differences, PSK which produced nearly equal times in each of two or more equally spaced phases would cause the vector sum to be nearly zero and synchronization to be not possible. Several methods of overcoming the difficulties are possible (Ref.3). One of these requires multiplication of the frequency of the signal by the modulus of the modulation (e.g., squaring for biphase keyed signals), so the modulation phases coincide and an unmodulated carrier is presented to the synchronizer. Unfortunately, the phase errors, frequency offsets and signal bandwidth are all multiplied too, so the range of time errors that the fine synchronizer can correct is reduced by the multiplication ratio. The effects of noise will also be increased by multiplication, which would require a compensating increase in signal-to-noise ratio at the synchronizer input. The narrowing of the synchronization range could be countered

by operating separately on an appropriate number of parts of the multiplied frequency band or by dividing the multiplied signal back to the original band. A possibly more attractive method than any of the above for removing the effects of phase modulation on synchronization is to use the recovered modulating waveform to restore an unmodulated carrier for the synchronizer. This method requires that it be possible to demodulate the signal before fine synchronization has been achieved, so some loss of demodulation efficiency is entailed.

Three classes of signal organization may be considered: many frequency hops per modulation state, one hop per modulation state and many modulation states per hop. The first assumes that the hopping rate is faster than the keying rate by a substantial amount. Since a large number of consecutive hops will be transmitted with the same modulation phase, suitable adjustment of the integration time of the synchronizer should permit synchronization with the same circuit used for unmodulated signals. The second signal form has modulation and hopping rates the same or nearly the same. The signal can only be demodulated by knowledge of a reference phase for each hop; this knowledge is obtained from fine synchronization, so synchronization must be accomplished before demodulation. The frequency multiplication method appears to be required for this signal form. The third signal form, many modulation states per hop, can be arranged for demodulation prior to fine synchronization by sending a reference phase at the start of each hop period or by using differential coding. Another possibility is the use of a set of orthogonal binary codes to represent the data states. Parallel correlators (one for each code) at the receiver would act as matched filters for the codes and enable recovery of the data (Ref. 4). Phase coherence is necessary only within the code length, and one or more code bursts would be sent during a single frequency hop period, so data receovery would not require that fine synchronization be accomplished. In a communication system using phase coherent frequency hop transmissions and the orthogonal code modulation, receivers at which no precise timing or range information was required could be simplified by omitting the fine synchronization capability. The cost of this flexibility is, of course, the increased modulation rate required (for a given data rate) by the codes.

FREQUENCY SHIFTS

Frequency offsets, from doppler effect or other causes, will interfere with fine synchronization more than with the earlier synchronization stages because the integration used in fine synchronization narrows the effective bandwidth of this process to a small fraction of the coarse synchronization bandwidth. The frequency multiplication method for removing PSK, if used, will make the synchronizer more sensitive to frequency offsets. It will be necessary to correct the frequency of the local reference to bring the input

to the fine synchronizer within the synchronizer passband. This is a problem which can be solved by conventional means. The coarse synchronization process is wide enough in bandwidth to accept the probable frequency shifts and to provide a preliminary frequency correction, so the fine synchronizer should have to correct only the remaining part of the frequency offset.

CONCLUSIONS

The fine synchronization method discussed in this report is sensitive to relatively small circuit perturbations and nonlinearities in the sense that they produce ripple in the detector response. The ripple degrades the accuracy of synchronization and may cause limited instability in the synchronization process. Available processing elements and techniques appear to have sufficient accuracy to permit the phase comparison technique to be used successfully in systems of moderate bandwidth, such as the experimental system with 18% bandwidth.

APPLICABILITY TO NAVY REQUIREMENTS

Precise relative navigation requires precise range measurements. These can be obtained from many signal waveforms. One waveform having advantages in some system configurations is frequency hopping, alone or in combination with code modulation. If the frequency hopping is phase coherent, then the fine synchronization method described here can enable precise range measurements to be made.

RECOMMENDATIONS

A study should be made to determine theoretically the relative merits of coherent frequency hopping systems and other systems capable of performing the same functions. The study should include the effects of noise, jamming and or propagation variations.

If the study shows coherent frequency hopping worthy of further development, and an experimental evaluation is desired, then a fine synchronizing method which avoids (or compensates for) the faults discussed in this report should be developed.

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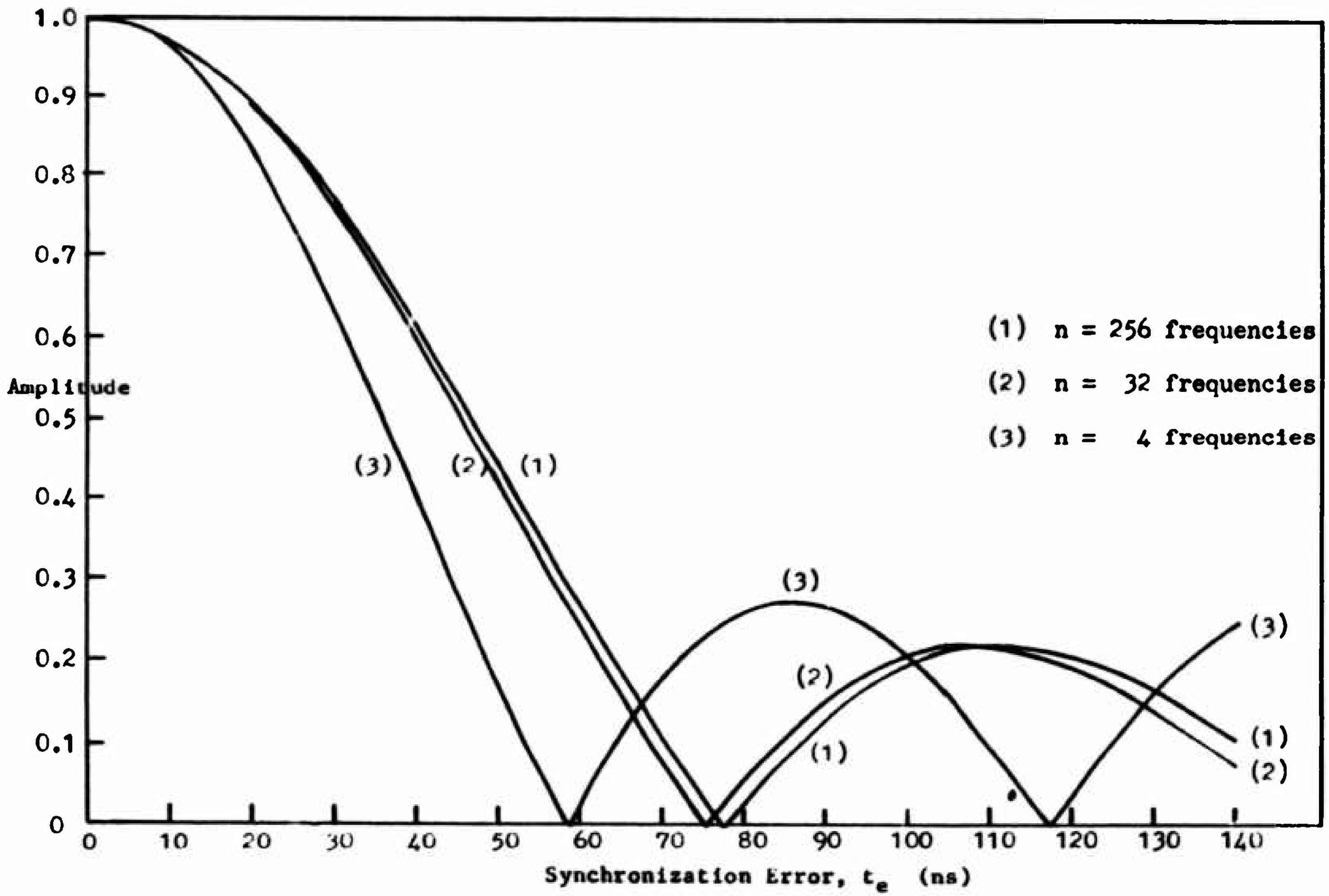

Fig. 1 — Integrated phase detector outputs as functions of synchronization error, for the indicated values of n

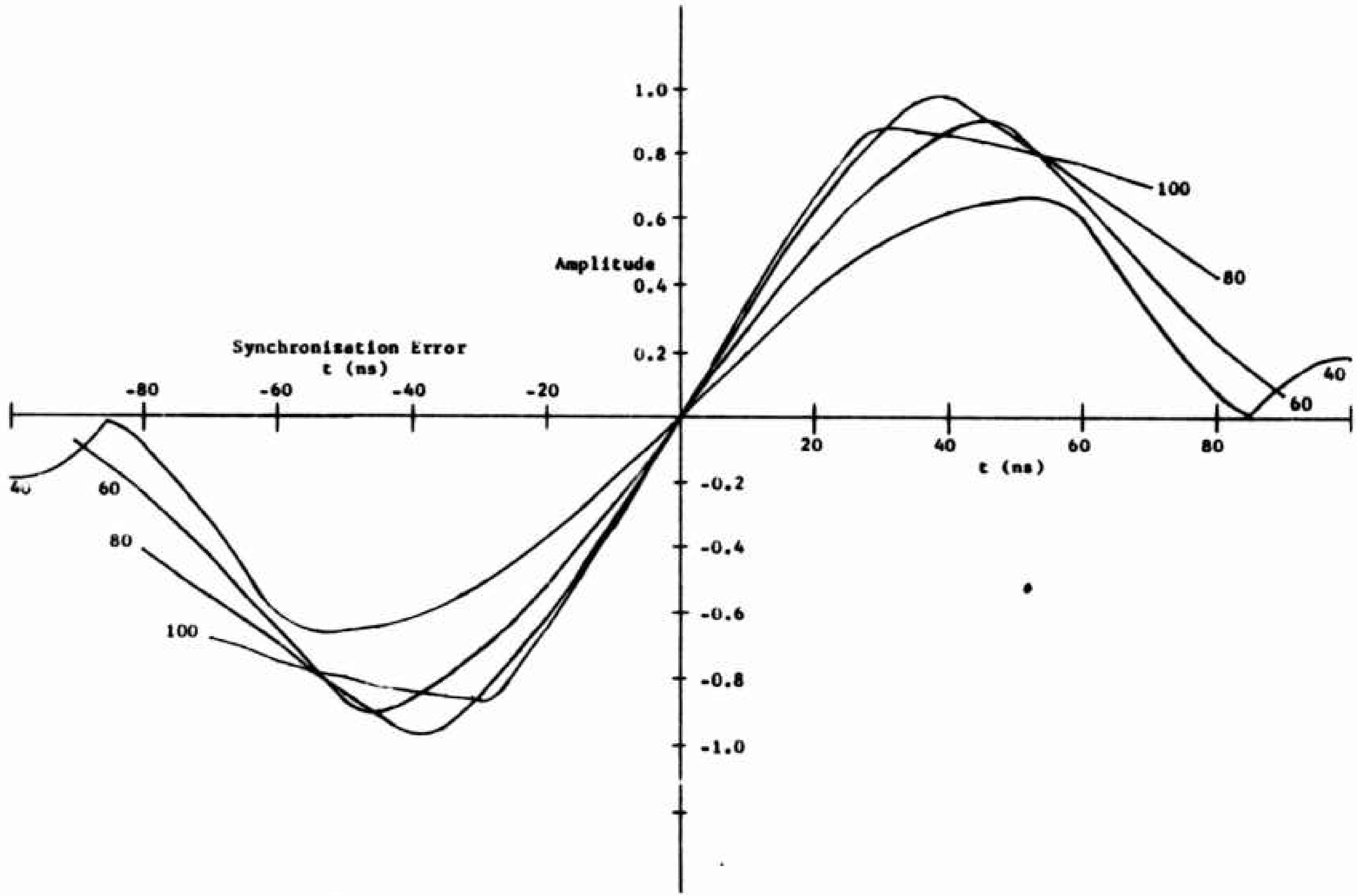

Fig. 2 — Difference values as functions of synchronization error, for the indicated values of d (in ns)

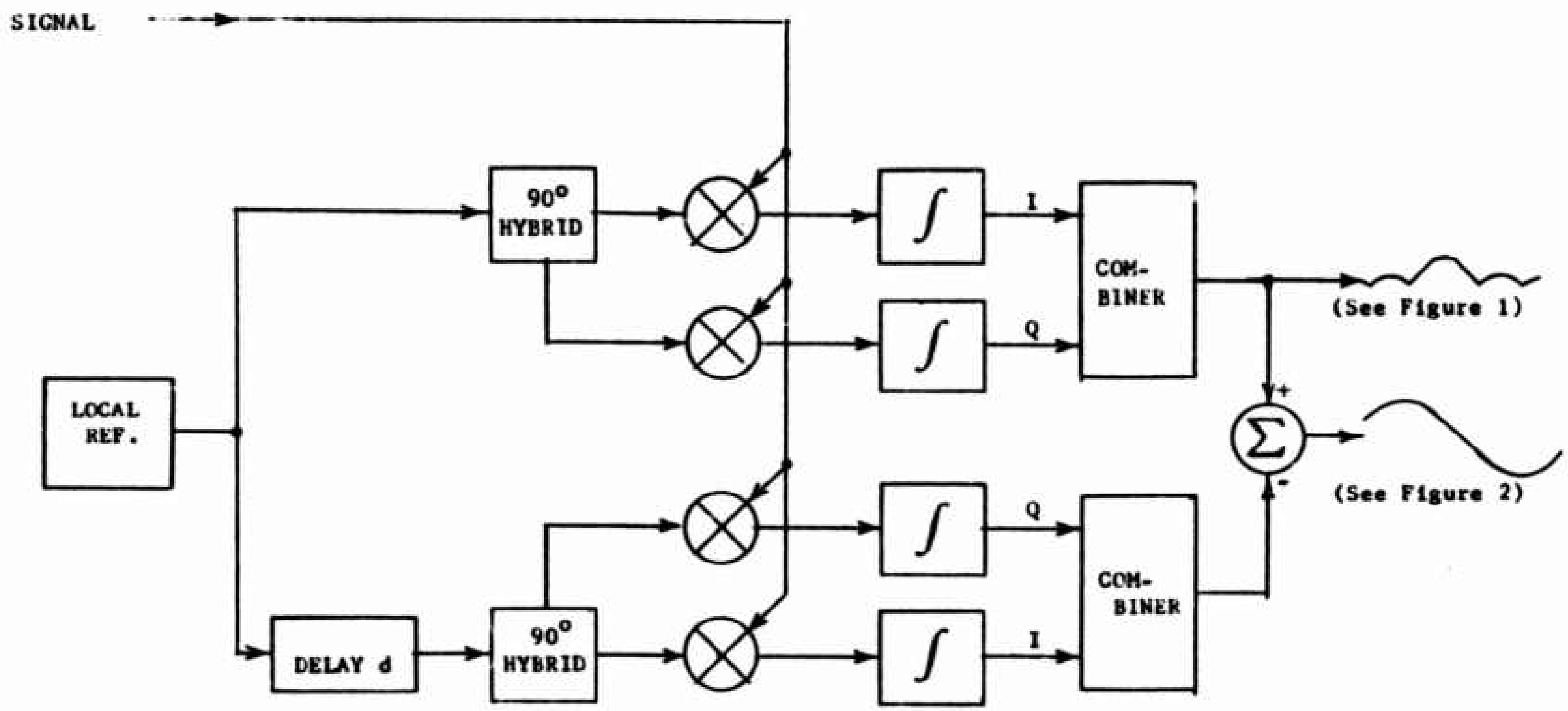

Fig. 3(a) — Fine synchronization error detector

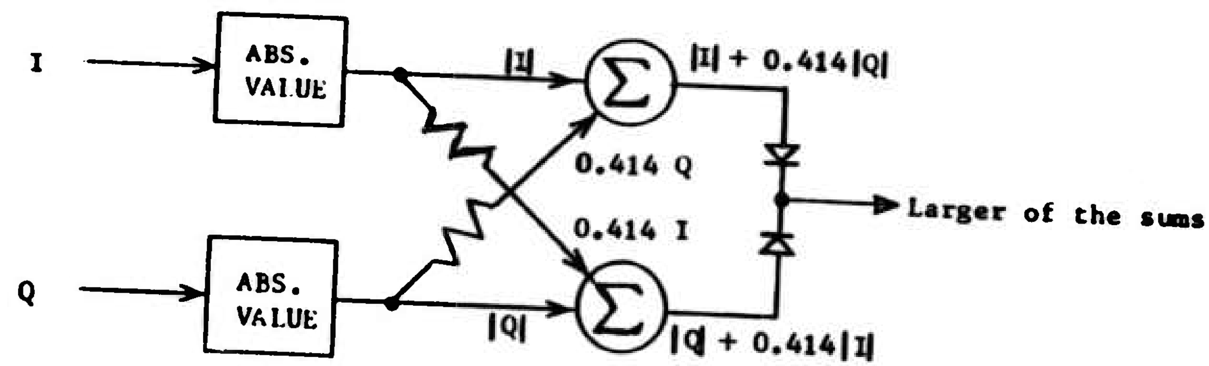

Fig. 3(b) — Approximate combiner

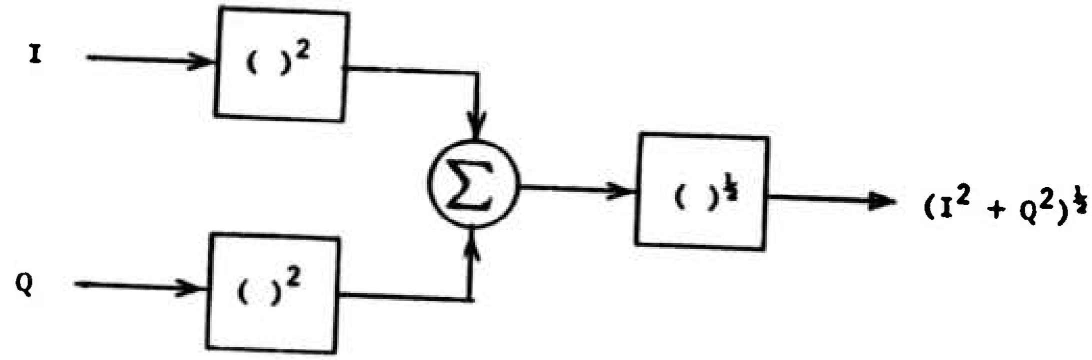

Fig. 3(c) — Exact combiner

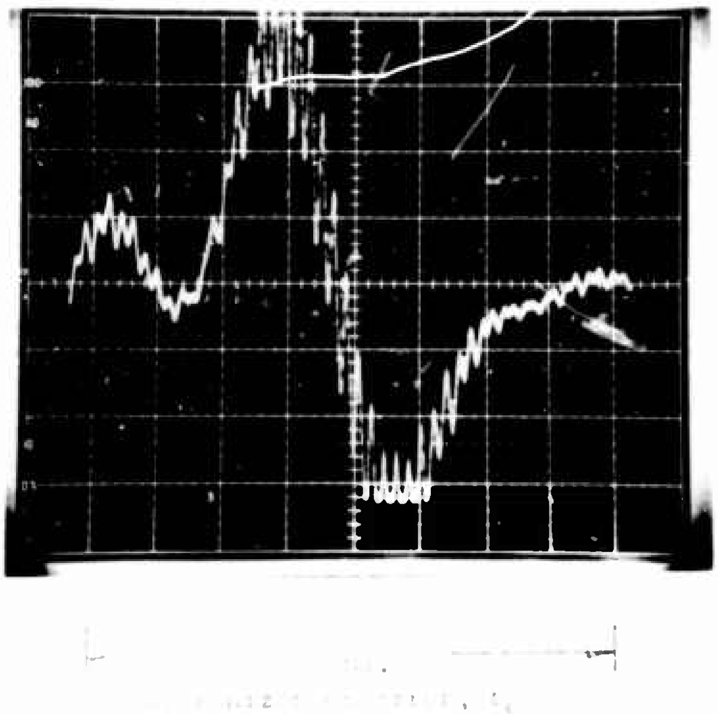

Fig. 4 — Difference output vs synchronization error

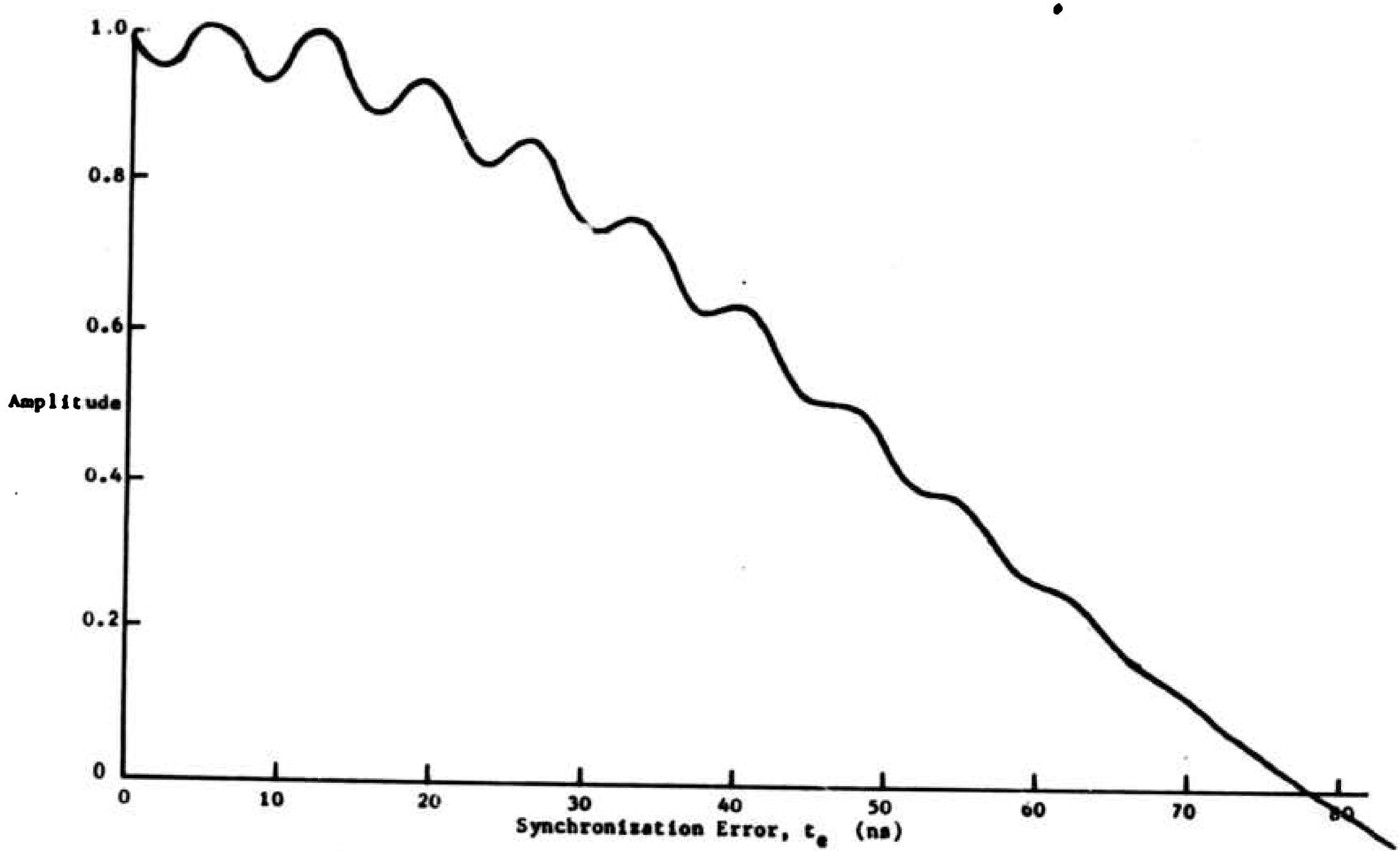

Fig. 5 — Integrated phase detector output for 5° quadrature error

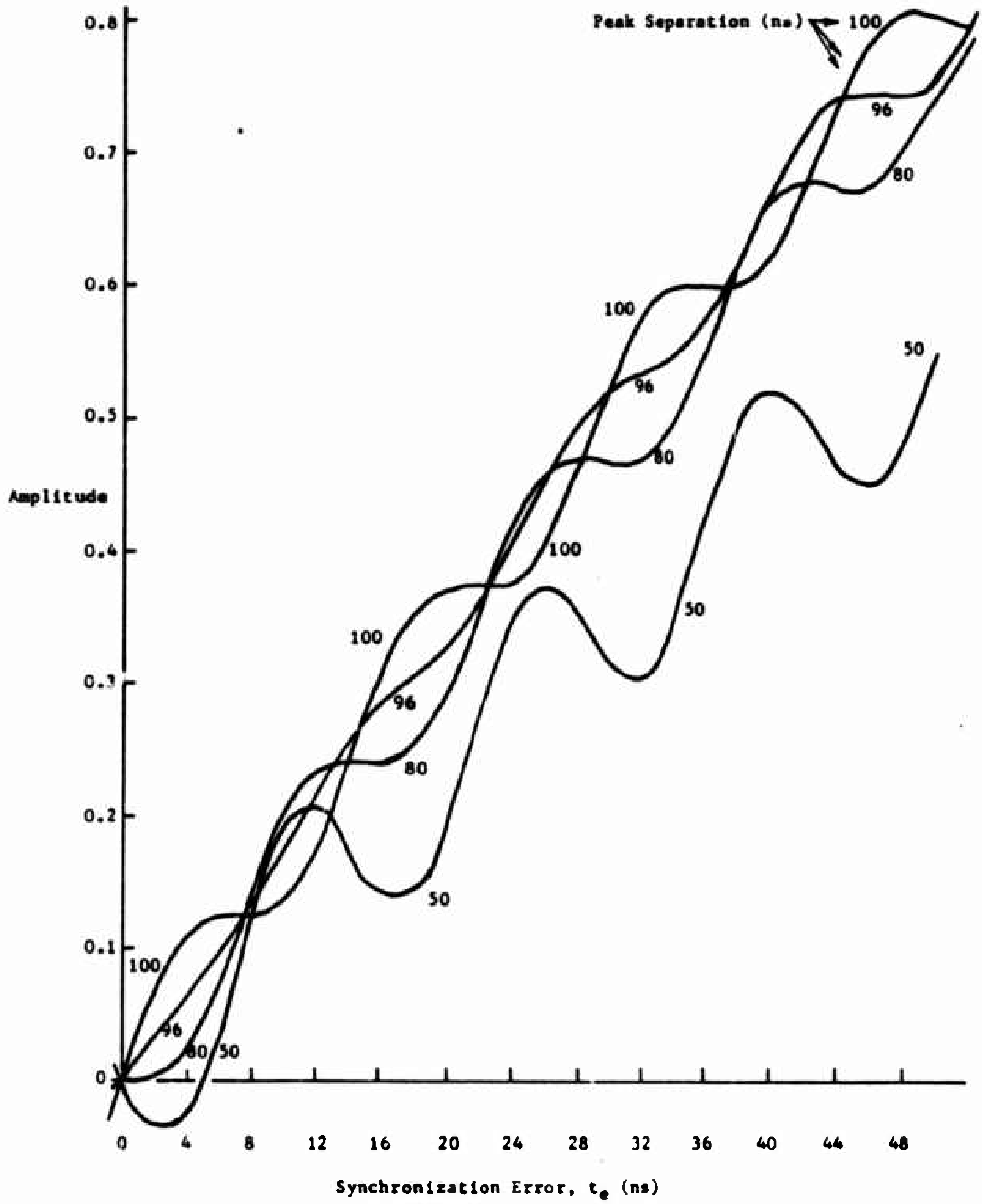

Fig. 6 — Difference values for 5° quadrature error

The maximum slope of the ripple component is derived from the general equation for R: $R = (1 - \sin 2a \sin b)^{\frac{1}{2}}$ (A1)

$$R^2 = 1 - \sin 2a \sin b \quad (A2)$$

$$2R(dR/da) = -2 \sin b \cos 2a \quad (A3)$$

$$dR/da = -(1/R) \sin b \cos 2a \quad (A4)$$

$$d^2R/da^2 = (1/R^2)(2R \sin b \sin 2a + \sin b \cos 2a \; dR/da) \quad (A5)$$

$$d^2R/da^2 = (1/R)(2 \sin b \sin 2a) - (1/R^3)(\sin^2 b \cos^2 2a) \quad (A6)$$

for maximum dR/da, set d^2R/da^2 equal to zero, then:

$$2R^2 \sin b \sin 2a = \sin^2 b \cos^2 2a \quad (A7)$$

$$2(1 - \sin 2a \sin b) \sin b \sin 2a = \sin^2 b \cos^2 2a \quad (A8)$$

$$2 \sin 2a - 2 \sin b \sin^2 2a = \sin b \cos^2 2a \quad (A9)$$

$$2 \sin 2a - \sin b \,(2 \sin^2 2a + \cos^2 2a) = 0 \quad (A10)$$

$$2 \sin 2a - \sin b \,(\sin^2 2a + 1) = 0 \quad (A11)$$

$$2 \sin 2a - \sin b \sin^2 2a - \sin b = 0 \quad (A12)$$

$$\sin^2 2a - (1/\sin b)(2 \sin 2a) + 1 = 0 \quad (A13)$$

A root of (A13) is: $\sin 2a = (1 - \cos b)/\sin b$ (A14)

Substituting from (A14) in (A4) to find the maximum dR/da,

$$dR/da)_{max} = \frac{-\sin b \cos 2a}{\left[1 - \sin b\left(\frac{1 - \cos b}{\sin b}\right)\right]^{\frac{1}{2}}} = \frac{-\sin b \cos 2a}{(\cos b)^{\frac{1}{2}}} \quad (A15)$$

$$\cos 2a = (1 - \sin^2 2a)^{\frac{1}{2}}, \text{ so} \quad (A16)$$

$$\cos 2a = \left[1 - \left(\frac{1 - \cos b}{\sin b}\right)^2\right]^{\frac{1}{2}} \quad (A17)$$

$$= \left[1 - \frac{1 - 2\cos b + \cos^2 b}{\sin^2 b}\right]^{\frac{1}{2}} \quad (A18)$$

$$= \left[\frac{\sin^2 b - \cos^2 b + 2 \cos b - 1}{\sin^2 b}\right]^{\frac{1}{2}} \tag{A19}$$

$$= \left[\frac{-2 \cos^2 b + 2 \cos b}{\sin^2 b}\right]^{\frac{1}{2}} \tag{A20}$$

$$\cos 2a = \left[(2 \cos b)(1 - \cos b)/\sin^2 b\right]^{\frac{1}{2}} \tag{A21}$$

$$= \left[(2 \cos b)\ (1 - \cos^2 \tfrac{1}{2}b + \sin^2 \tfrac{1}{2}b)/\sin^2 b\right]^{\frac{1}{2}} \tag{A22}$$

$$= \left[(4 \sin^2 \tfrac{1}{2}b \cos b)/\sin^2 b\right]^{\frac{1}{2}} \tag{A23}$$

$$\cos 2a = \frac{2 \sin \tfrac{1}{2}b}{\sin b} (\cos b)^{\frac{1}{2}} \tag{A24}$$

substituting (A24) in (A15),

$$dR/da)_{max} = \frac{- \sin b \dfrac{2 \sin \tfrac{1}{2}b}{\sin b} (\cos b)^{\frac{1}{2}}}{(\cos b)^{\frac{1}{2}}} = -2 \sin \tfrac{1}{2}b \tag{A25}$$

The phase angle, a, shifts through a complete cycle for a synchronization error, t_e, shift of one period of the frequency applied to the mixers of Figure 3A. In the experimental system the frequency is 70 MHz, so a timing shift of 1/0.07 nanosecond will shift angle a 2π radians. Restated, the relationship between phase angle a and synchronization error t_e in the experimental system is:

$$1 \text{ ns} = 2\pi \times 0.07 = 0.4398 \text{ radians} \tag{A26}$$

so:

$$dR/dt_e)_{max} = -0.8796 \sin \tfrac{1}{2}b \quad \text{units/nanosecond} \tag{A27}$$